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Brief Communication

Velocity of long bubbles in horizontally oscillating vertical pipes

J. Kubie*

Department of Mechanical, Manufacturing & Systems Engineering, Napier University, Edinburgh, EH10 5DT, Scotland, UK

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1. Introduction

The velocity of large gas bubbles in liquids has been investigated by many authors, both theoretically and experimentally. A recent comprehensive review (Clarke and Issa, 1993) indicates the latest analytical developments. The results for the rise velocity U_0 of long gas bubbles in stagnant liquids in vertical pipes are

$$U_0 = k_0(gD)^{0.5} \tag{1}$$

where g is the gravitational acceleration, D is the internal pipe diameter and k_0 is a dimensionless parameter, which depends primarily on the viscosity and the surface tension of the liquid and the internal diameter of the pipe. Extensive experimental work by Zukoski (1966) indicates that if the pipe is sufficiently large the effects of the physical properties of the liquid are negligible and the dimensionless parameter is constant and given by

$$k_0 = 0.35 \tag{2}$$

In many industrial situations when long bubbles or slug flows occur, the pipes undergo severe vibrations which affect the rise velocity of the long bubbles. Vibrations of vertical pipes have been observed in many applications. Vertical vibrations along the axes of the vertical pipes occur when slug flow from these pipes discharges into partially filled horizontal tanks. Standing

* Tel.: 0131-455-2595; fax: 0131-455-2264.
E-mail address: j.kubie@napier.ac.uk (J. Kubie).

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waves on the interface in the tank can then result in vertical vibrations of the tank and the attached pipes.

The rise velocity of long bubbles in vertical pipes which vibrate vertically along their axes have been recently investigated by Brannock and Kubie (1996), who subjected the vertical pipe to a sinusoidal vertical motion, with amplitude A and angular velocity ω. Their experimental results showed that it is primarily the relative acceleration a defined as

$$a = A\omega^2/g \tag{3}$$

which influences the bubble rise velocity U, and that the rise velocity is significantly reduced as the relative acceleration increases. This observation agreed with previous results which indicate that settling or rise velocity of particles or small bubbles in liquids can be considerably reduced if the liquid is subjected to vertical oscillations (Boyadzhiev, 1973; Herringe, 1976; Kubie, 1980).

Even though the vertical vibrations of vertical pipes are important, it is the horizontal vibrations of such pipes which are much more common. Horizontal vibrations across the axes of vertical pipes occur in many situations, such as in certain types of shell-and-tube heat exchangers, when one of the fluids flows across the axes of the vertical pipes. This can then be accompanied by vortex shedding, which can then easily induce horizontal oscillations.

It is the purpose of this paper to investigate the rise velocity of large gas bubbles in horizontally oscillating vertical pipes, by subjecting the pipes to a sinusoidal horizontal motion

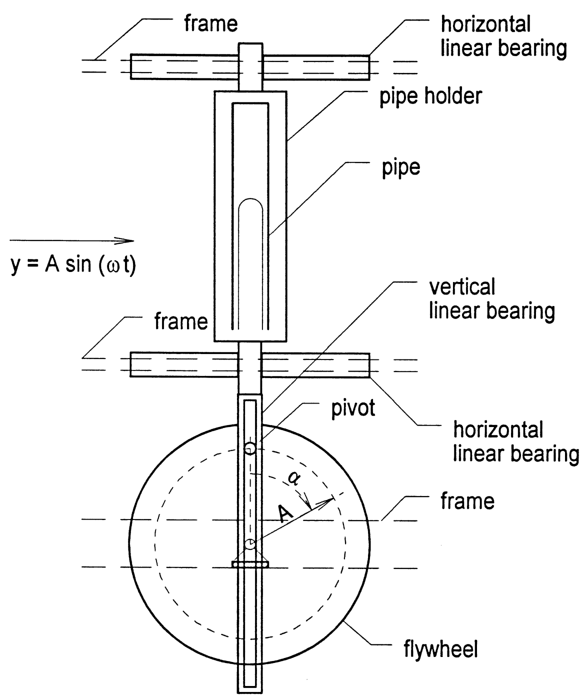

Fig. 1. Diagram of the experimental apparatus.

perpendicular to their vertical axis, and to compare the results with those obtained for the vertically oscillating vertical pipes.

2. Experimental work

An apparatus has been designed which enables movement of a vertical pipe in an exact sinusoidal horizontal motion. The apparatus is a modified version of the apparatus used in the previous study (Brannock and Kubie, 1996). The apparatus is sufficiently flexible to allow for changes in the amplitude and the frequency of the angular motion. A diagram of the experimental apparatus is shown in Fig. 1. The major components are: a rigid frame, flywheel, scotch yoke mechanism and bearings, vertical pipe holders and vertical pipes. A flywheel, powered by an electric motor via a gearbox, was used to vary the amplitude of the horizontal motion and to improve the dynamic stability of the rig. In order to obtain a sinusoidal horizontal motion a Scotch yoke mechanism was used. A pivot, connected to the flywheel, was inserted into a roller bearing. The position of the pivot determined the amplitude of the horizontal motion. The roller bearing was then bolted onto a sliding member of a vertical linear bearing, which was connected to the pipe holder. An arrangement of two horizontal linear bearings fixed to a rigid frame was used to keep the pipe holder vertical. As the flywheel started to spin a horizontal sinusoidal motion of the vertical pipe was obtained. The rigidity of the apparatus was checked by estimating qualitatively its natural frequency. The excitation frequencies were always well below the natural frequency of the structure.

Three perspex pipes, 2000 mm long, with internal diameter D of 22, 44 and 52 mm, and sealed at the top, were used in the experiments. Prior to the experimental work, the pipes were cleaned with water. The pipes were filled with the working fluid, tap water at room temperature, and closed with a stopper. The influence of the following parameters of the sinusoidal motion were investigated: amplitude A of 50, 100 and 140 mm and acceleration $A\omega^2$ of 0 (equivalent to a stationary vertical pipe), 2, 4, 6, 8, 10 and 12 m s^{-2}. The apparatus did not allow higher accelerations, since significant vibrations started for accelerations above 12 m s^{-2}.

The required amplitude A and angular velocity ω of the sinusoidal motion were set and the electric motor was started. Steady-state conditions were quickly reached, as determined by measuring the angular velocity as a function of time. When the steady-state conditions were reached the stopper was released and a long air bubble started to rise in the vertical pipe. Different ways of removing the stopper were investigated but they had no noticeable effect on the rise velocity of the bubbles. The pipes were graduated at 200 mm intervals, and elapsed times between the intervals, as well as the total times taken for the bubbles to rise between 300 and 1900 mm, were taken with a stopwatch. The former times were taken to determine the bubble rise velocity over 200 mm length of the pipe U_I and the latter times to determine the average bubble rise velocity $\bar{U}$ over the whole pipe. Whereas the determination of the velocity U_I was subject to considerable error of up to 20%, it is estimated that the maximum error in determining the average bubble velocity was less than 5%. In order to decrease this error further, each experimental run was repeated five times and the average bubble rise velocity over the five runs was calculated. It is estimated that the maximum error is then much less than 5%. Finally, photographs of the bubbles were taken to study the shapes of the bubbles.

3. Results

The bubble rise velocity in stationary pipes U_0 is in general agreement with the results of previous studies. However, in the present work the constant k_0 was measured as 0.34, rather than 0.35, obtained by Zukoski (1966). In order to preserve consistency, the experimental values of the present work will be used when the velocity U_0 is required.

Typical experimental results for the variation of the velocity U_I along the largest diameter pipe (D = 52 mm) are plotted for A = 140 mm and three different values of the relative acceleration a in Fig. 2. It should be pointed out that for this diameter pipe the bubble rise velocity in the stationary vertical pipe was obtained experimentally as $U_0 = 0.24$ m s^{-1}. Two typical observations can be made from Fig. 2. First, the velocities U_I are subject to considerable fluctuations, and these fluctuations would have been much more pronounced if the velocities were determined over much shorter intervals. Indeed, the instantaneous bubble rise velocities do fluctuate considerably. The reason for these fluctuations is primarily the magnitude and the direction of the resultant acceleration (discussed below), which affects the bubble shape and controls the instantaneous bubble rise velocity. The fluctuations in the velocity U_I are well below the qualitatively observed fluctuations of the instantaneous bubble

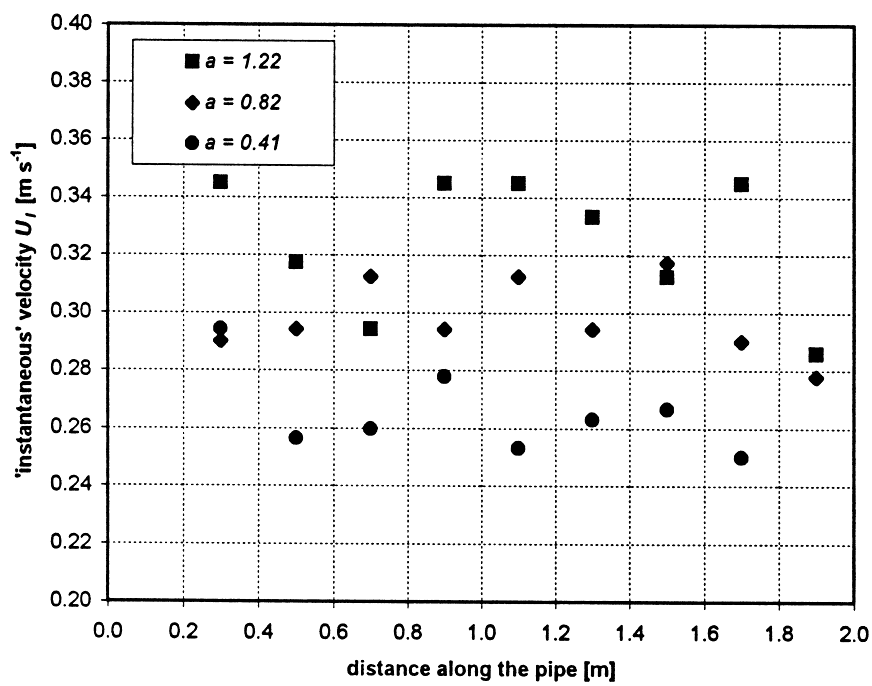

Fig. 2. The effect of the relative acceleration on the variation of the bubble rise velocity U_I along the largest diameter pipe (D = 52 mm, A = 140 mm, $U_0 = 0.24$ m s^{-1}).

rise velocity. The reason is that during the bubble rise over the distance of 200 mm the system experiences at least four cycles and hence the instantaneous fluctuations are smoothed out. Second, as the relative acceleration a increases the velocities U_I also increase, and they lie well above the bubble rise velocity in the stationary pipe.

Whereas for a given diameter pipe and given oscillatory conditions the instantaneous bubble rise velocities and velocities U_I fluctuate considerably, the corresponding average bubble rise velocity $\bar{U}$ is practically constant (typically, for $\bar{U} = 0.25$ m s^{-1}, the standard deviation was determined as 0.005 m s^{-1}). The ratios of the average bubble rise velocity and the bubble rise velocity in the stationary pipe $\bar{U}/U_0$ is plotted against the relative acceleration a for the three pipe diameters D and three different amplitudes A = 50, 100 and 140 mm in Figs. 3–5 respectively.

When the pipe was stationary the bubbles adopted the typical axisymmetric shape observed previously (Zukoski, 1966). However, as the relative acceleration increased, the shape of the bubble nose became asymmetric and changed periodically in line with the frequency of the sinusoidal horizontal motion imposed on the pipe. Furthermore, the asymmetry of the shape of the bubble nose increased with increasing relative acceleration a. Typical shapes of the bubble nose, obtained as idealised tracings of the photographs, are shown for the D = 52 mm diameter pipe and three different conditions in Figs. 6–8. Fig. 6 shows the shape of the bubble nose in a stationary pipe; Fig. 7 shows the shape of the bubble nose for A = 50 mm and a = 0.41, and Fig. 8 shows the shape of the bubble nose for A = 50 mm and a = 1.02. It should be further noted that the investigation of the shapes of the bubbles is beyond the scope

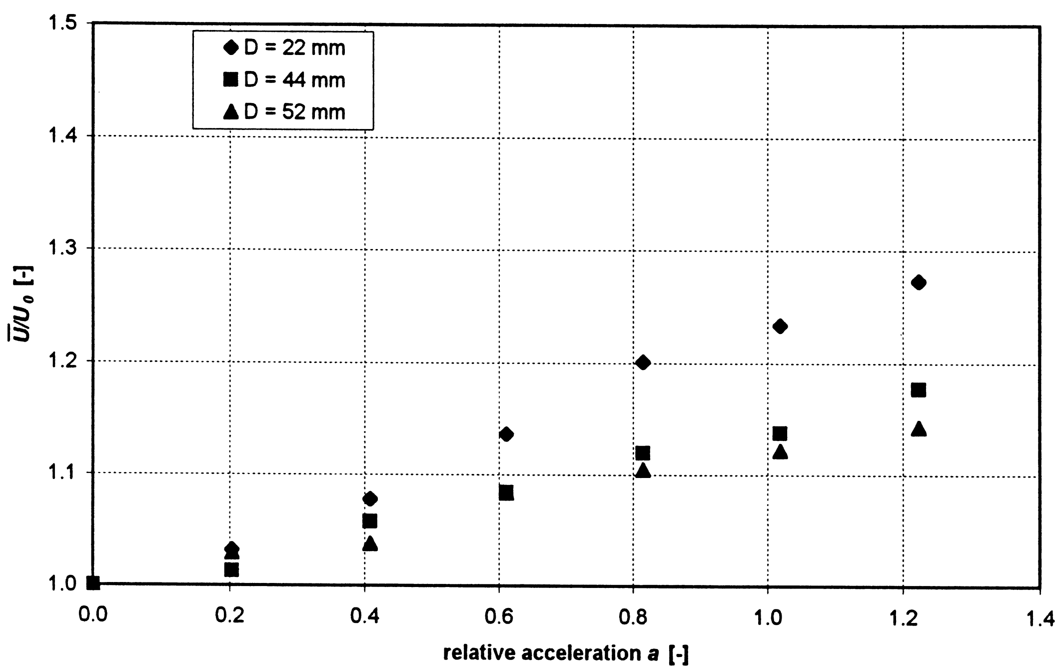

Fig. 3. The effect of the pipe diameter on the variation of the velocity ratio with the relative acceleration, for A = 50 mm.

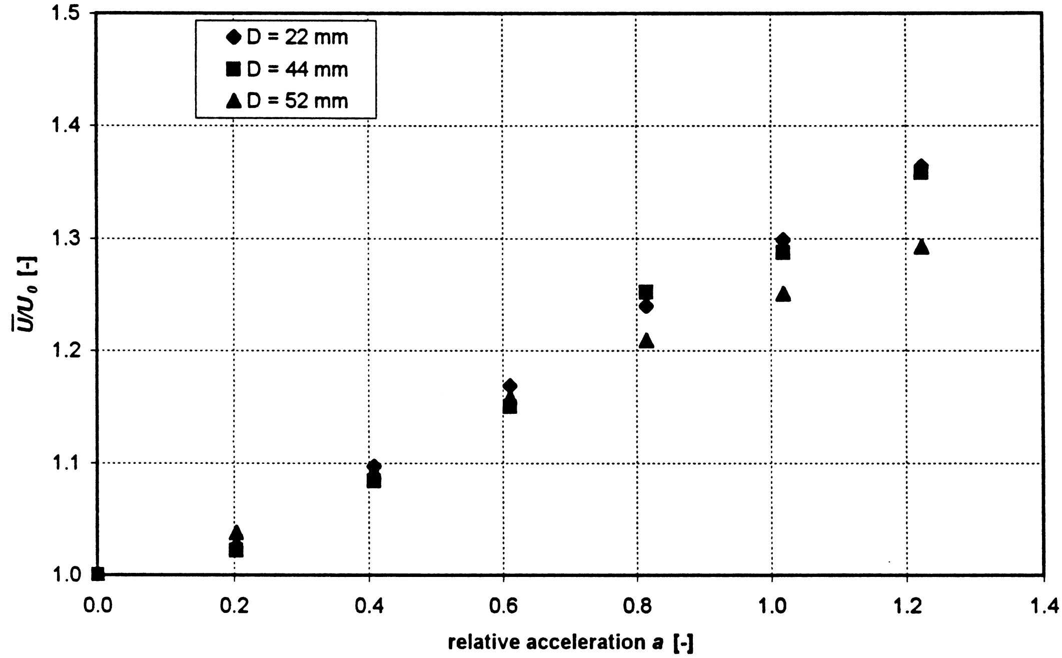

Fig. 4. The effect of the pipe diameter on the variation of the velocity ratio with the relative acceleration, for A = 100 mm.

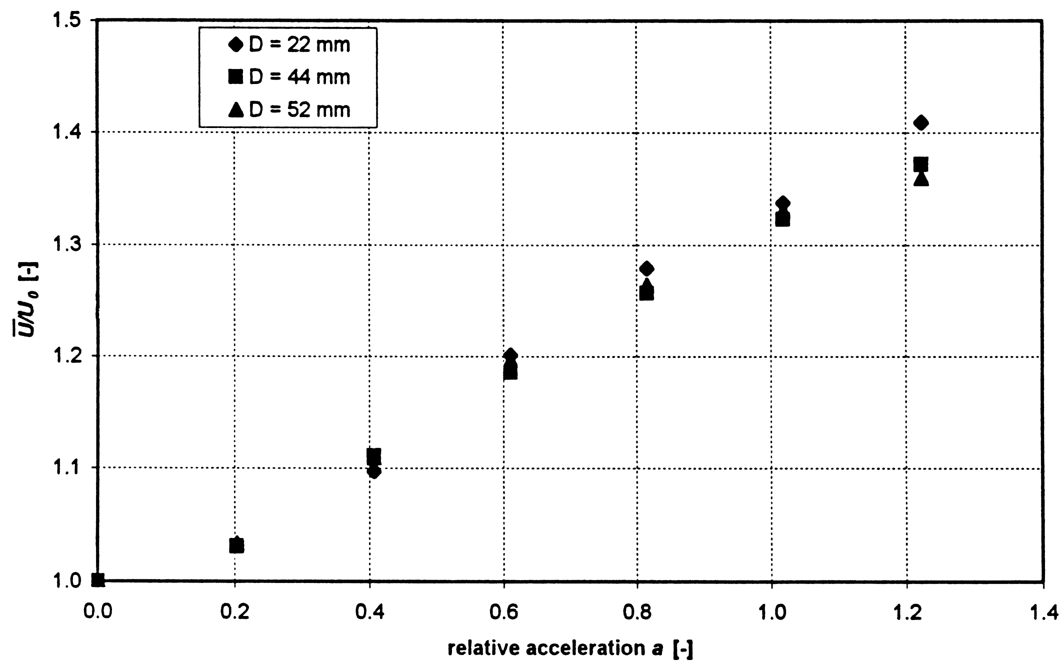

Fig. 5. The effect of the pipe diameter on the variation of the velocity ratio with the relative acceleration, for A = 140 mm.

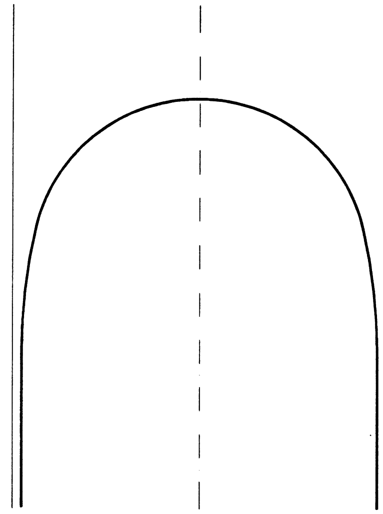

Fig. 6. Bubble geometry in a stationary pipe (D = 52 mm).

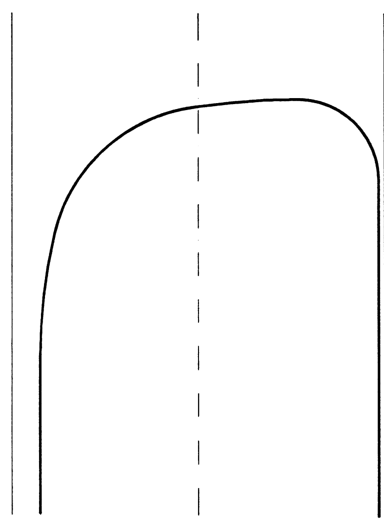

Fig. 7. Typical bubble geometry in an oscillating pipe (a = 0.41, D = 52 mm, A = 50 mm, $\alpha \approx 110°$).

of the present work and that the idealised tracings do not take into account either the second order variations in the film thickness or the optical distortions.

4. Discussion

The shapes of the bubble noses in oscillating pipes are remarkably similar to the shapes observed by Zukoski (1966) in his investigation of the bubble rise velocity in inclined pipes.

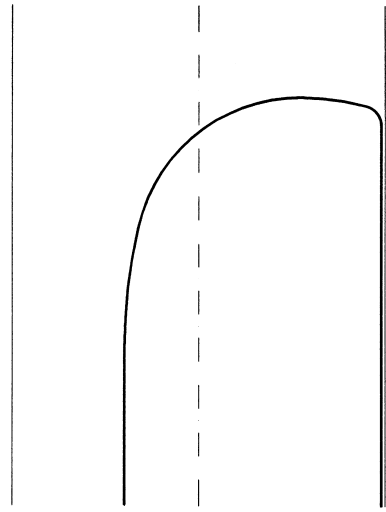

Fig. 8. Typical bubble geometry in an oscillating pipe (a = 1.02, D = 52 mm, A = 50 mm, $\alpha \approx 110°$).

That the similarity is more than just coincidental can be demonstrated by the following examination of the accelerations (and hence forces) acting on the pipe, and the enclosed gas bubbles and water.

Consider Fig. 9 which shows the accelerations acting on the system: the gravitational acceleration g acting in the vertical direction, and the acceleration $A\omega^2 \sin(\omega t)$ due to the oscillation of the pipe and acting in the horizontal direction. The resultant acceleration r will be at an angle β to the horizontal, where β is given by

$$\beta = \tan^{-1}\frac{g}{A\omega^2 \sin(\omega t)} \tag{4}$$

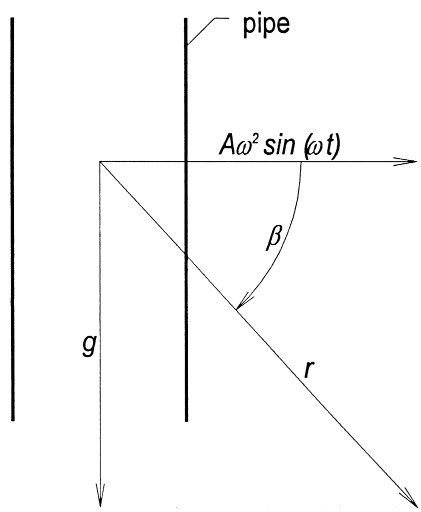

Fig. 9. Diagram of the accelerations acting on the system.

For the angle of the pivot α between 0 and π, the angle β will be given by

$$\beta_M \le \beta \le \pi/2 \tag{5}$$

where

$$\beta_M = \tan^{-1}\frac{1}{a} \tag{6}$$

Hence the minimum angle of the resultant acceleration with the horizontal is only a function of the relative acceleration a and it decreases with the increasing relative acceleration. For the present experimental work the minimum angle β_M varies between 90° (for a stationary vertical pipe) and 39° for the highest relative acceleration of 1.22.

If the resultant acceleration were constant, rather than varying with time, and equal to the maximum acceleration, the present system would be physically equivalent to a gas bubble rising in a stationary pipe inclined at an angle β_M to the horizontal, or in exactly the same system as investigated by Zukoski (1966). This can be confirmed by noting that the distortion of the bubble nose increases as the relative acceleration increases (and the angle β_M decreases) which is in agreement with the experimental observations of Zukoski (1966).

Although the two systems are qualitatively equivalent, there is one major difference. Whereas the system investigated by Zukoski (1966) is stationary and the angle of pipe inclination is constant, the present system is highly unsteady, and the equivalent angle of inclination will vary between $180°-\beta_M$ and β_M during each period of oscillation. This implies that in the system investigated in the present work the relative position of the bubble changes: the bubble moves periodically from one side of the pipe to the other. Nevertheless, it is also possible to compare the two systems quantitatively.

Zukoski (1966) showed that as the angle of the pipe inclination decreases from the vertical, the bubble rise velocity increases, reaches its maximum for the angle of inclination of about 40° and then starts decreasing. He further showed that for a given liquid, the maximum bubble rise velocity is more pronounced as the pipe diameter increases. Finally, his experimental results show that for air and water the ratio of the bubble rise velocity in a pipe inclined at 45° to horizontal and a bubble rise velocity in a vertical pipe is 1.38, 1.58 and 1.62 for the pipe diameters of 22, 44 and 52 mm, respectively. These results can be compared with the experimental results presented in Figs. 3–5.

Figs. 3–5 show that as the relative acceleration increases (and the equivalent angle of the pipe inclination decreases towards 40°), the velocity ratio $\bar{U}/U_0$ increases. The figures also show that for a given amplitude A of the horizontal sinusoidal motion the velocity ratios increase with the decreasing diameter of the pipe. Furthermore, as the amplitude increases the velocity ratios increase too, but even the maximum velocity ratio for D = 22 mm and the relative acceleration a = 1 (which is equivalent to $\beta_M = 45°$) is below the value of 1.38 discussed above. The reasons for this observation are probably two-fold. First, as pointed out above, the equivalent angle of inclination is not constant and during each period the position of the pipe changes from inclined to one side, vertical and inclined to the other side. Hence, apart from the inertia and possible additional mechanisms due to the unsteady motion, the pipes will remain for a fraction of the total time in considerably less inclined positions. Second, when the amplitude A is comparable with the pipe diameter D, the inertia effects will become more pronounced and thus the equivalent *effective* inclination will be comparatively smaller as the ratio A/D decreases.

Finally, a comparison can be made between the results for the vertically oscillating vertical pipes (Brannock and Kubie, 1996) and the horizontally oscillating vertical pipes. In the former case, the velocity ratio decreases as the relative acceleration increases, but in the latter case the velocity ratio initially increases as the relative acceleration increases. Furthermore, in the former case it is primarily the relative acceleration and possibly the pipe diameter, and not the amplitude, which govern the velocity ratio, but in the latter case, all three parameters govern the velocity ratio.

The analysis presented indicates that it is the resultant acceleration which primarily influences the behaviour. In the former case, when the acceleration due to the oscillation of the pipe is parallel with the gravitational acceleration, the resultant *effective* acceleration is lower than for a stationary pipe and parallel with the axis of the pipe. However, in the latter case, when the acceleration due to the oscillation of the pipe is perpendicular to the gravitational acceleration, the resultant *effective* acceleration is higher than for a stationary pipe, but, and more importantly, at an angle to the axis of the pipe. Hence in the former case the vertical oscillations decrease the effective acceleration in a vertical pipe, thus decreasing the rise velocity of the bubbles, but in the latter case the horizontal oscillations have the effect of inclining the pipe, thus, at least initially, increasing the rise velocity of the bubbles.

5. Conclusion

The rise velocity of long bubbles in vertical pipes subjected to a sinusoidal horizontal motion has been investigated. It has been shown that the average bubble rise velocity is a function of the relative acceleration a, the amplitude of the sinusoidal motion A and the diameter of the pipe. The average rise velocity increases, at least initially, with the relative acceleration. This increase in the average bubble rise velocity is compared and contrasted with the *decrease* in the average bubble rise velocity in vertical pipes subjected to a sinusoidal *vertical* motion. It is shown that the reason for this fundamental difference is the magnitude and direction of the resultant acceleration acting on the bubbles.

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